Astronomical Observations and Architecture at Monte Albán, Teotihuacan, and Stonehenge

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Abstract

It is well established that Mesoamerican cultures employed a complex calendar based on two cycles, the 365 day solar cycle and a 260 day ritual calendar. Peeler and Winter have reported that the ratio of these two numbers is enshrined in unique features of the architecture at both Monte Albán in the Oaxaca Valley and Teotihuacan in the Valley of Mexico.

We report here further investigations on the uniqueness of this ratio at these sites. The measurements reported by them are capable of supporting a multitude of different ratios, with errors a hundred-fold smaller than found for the 365/260 ratio.

We have investigated the architecture of a site remote from Mesoamerica by over 2500 years and more than 8500km. At this site (Stonehenge in Wiltshire, England), there is sufficient architectural evidence for a detailed statistical analysis, and we find that the ratio 365/260 defines the basic structure of the first stones erected at the 5σ level–about a million to one against this happening by chance.

These results indicate that the importance of the 365/260 ratio was recognized long before it appeared in Mesoamerica, that the number 260 was independent of latitude, and could not have originated in observations of zenith and nadir passages of the sun at Monte Albán. We show that the origin of the number 260 could have arisen from observations of the Moon and the planet Venus, and must have long preceded religious interpretation by the inhabitants of Mesoamerica.

We report evidence that the synodic period of the planet Jupiter was also important in the design of the earliest structure at Stonehenge.

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1. Introduction

This investigation was prompted by reports by Peeler and Winter [WINTER95] and recently expanded by them [WINTER10]. The argument of Peeler and Winter is based upon finding that the ratio of two measured distances is, at least approximately, equal to the ratio of two integers. Obviously there will be some error limit defining any such identity. This of course is a very reasonable approach, it is likely that the builders would measure out distances using an integral number of units–strides for example. It is very unlikely that they would have had any concept of a fractional system (such as our decimal system). The second step in the argument involves an identification with the same integral ratio found elsewhere; in this case with the ratio 365/260 (=73/52) of the two calendar systems. However, if the measured distances should happen to approximate to more than one ratio of integrals, the appropriate procedure would be to compare all the integral ratios found, in order of decreasing error margin, best fit first, with ratios arising from other sources and deemed important. In this paper we set an error limit just wide enough to include the ratio 365/260 and find that for all the measurements reported by Peeler and Winter very many integral ratios are found, with errors sometimes hundreds of times lower than that for 365/260. In this situation, the identity with 365/260 depends upon setting the error margin wide enough to include 365/260, and the process reduces to increasing the error margin until you find identity with the ratio you are looking for. Reduce the error margin and the identity disappears.

Firstly Peeler and Winter proposed that ratios of certain dimensions at Monte Albán approximate to the ratio of the number of days in the solar (365) and the Mesoamerican ritual (260) calendars. They also noted that the location of the Zapotec Barrio (Tlailotlacan) at Teotihuacan relative to the Pyramids of Quetzalcoatl and the Moon exhibits this same ratio. Secondly, and independently of these measurements of lengths, although no particular structure within the Mont Albán site is, with any certainly, oriented to either the zenith or nadir passage of the sun, they did identify a line at approximately 108° from Building J to Building O at Caballito Blanco, 35km away in the Tlacolula Valley. This is the line of the nadir passage on August 8. Unfortunately, as they note, the line of sight between J and O is totally obscured by the intervening peak of Cerro Yani Grande. Furthermore, the direction of the nadir sunrise in the east at 108° can only be indirectly observed by the sunset in the west at 288° on the day of the nadir. This line of sight from Building J would be obscured by Building M if this happened to pre-date Building J.

They also noted that a point in the Teotihuacan site could be chosen such that the two sight lines over the pyramids of Quetzalcoatl and the Moon define an angle close to the angular separation (36°) between the zenith and nadir sunrises at Monte Albán. One such point occurs within the Zapotec barrio at Tlailotlacan, and they noted that the ratio of the distance from this point to the pyramid of Quetzalcoatl to that of the separation between the two pyramids was close to 365/260. With the further restriction that the line of sight from Tlailotlacan over Quetzalcoatl must be approximately 108°, the point in Tlailotlacan is the only solution. However, this solution requires that the lines of sight over the pyramids do *NOT* coincide with the zenith and nadir passages of the sun. The angular separation of the zenith and nadir passages is quite strongly dependent upon latitude and is 42° at Teotihucan.

On this basis they propose that this angular separation of 36° was transported by Zapotecs from Monte Albán to Teotihuacan along with the importance of the ratio 365/260. This implies that the angle of 36° was more important to the Zapotec immigrants at Teotihuacan than any of the solar events which might have defined that angle at Mont Albán–a *sacred* angle. It would be more convincing if the 36° angle was indeed incorporated into the architecture at the Monte Albán site.

They conclude that aspects of the architecture of both Monte Albán and Tlailotlacan at Teotihuacan were deliberately designed to reproduce the 108° and the 36° angles together with the ratio 365/260.[i] Finally they tentatively suggest that the Zapotec culture in the Valley of Oaxaca might have been the origin of the 260 day religious cycle used by the Mayans of Mesoamerica. The question remains open as to the choice of 260 days for the religious cycle.

Combining the 365/260 ratio and the angular separation of zenith passages with the orientation (15°28' east of north) of the Street of the Dead (joining the Temples of Quetzalcoatl and the Moon) they stress that the triangle completed by the location of Tlailotlacan is invariant and unique to the latitude of Teotihuacan. Peeler and Winter define the conditions for their rigid triangle at Teotihuacan to be

> *This triangle, with*
>
> - *sight lines oriented to the two sunrises on important Zapotec zenith passage and nadir passage dates,*
> - *proportioned 260 to 365, and*
> - *the 260-day line perpendicular to the two identical-azimuth sunsets separated by 260 days*
>
> *can exist only at a latitude of 19°41'–the latitude of Teotihuacan.*
>
> —Damon E Peeler and Marcus Winter, *Sun Above, Sun Below, [WINTER10] (p.22)*

A problem lies with the first condition, the sight lines. The sight line from Tlailotlacan over the pyramid of Quetzalcoatl (107°1.5') and that over the pyramid of the Moon (71°1.5') do not coincide with the zenith passages of the sun at 111° and 69°. The discrepancy is of magnitude 2° (4 sun widths) at the Moon and 4° (8 sun widths) at Quetzalcoatl. There are two possible interpretations: either the Zapotecs at Tlailotlacan were very badly in error in establishing the sight lines over the pyramids, or, they simply transfered a *sacred angle of 36°* from Monte Albán to Teotihuacan without any understanding of its relevance to the passages of the sun. I find it difficult to accept either solution. If we accept the locations of the pyramids and the angle at Tlailotlacan (all observable today), then there are several locations for Tlailotlacan which satisfy both the 36° angle and the 365/260 ratio, but none of them provide lines of sight over the pyramids to the zenith and nadir passages of the sun. We explore the existence of these sites below.

There is a relevant but independent problem in the origin of the 260 day period used in the religious calendar of the Mayan peoples. No-one so far has been able to find a convincing

[i] They also noted instances of a size ratio close to the Venus cycle, 584/365, at both sites. We discuss this further below.

astronomical origin, so we are left with considerations of place (e.g. latitude), culture (e.g. religion), and possibly time (e.g. one unique time) Peeler and Winter list the possible origins suggested for the choice of 260, and add another based on their rigid triangle approach, but none are particularly convincing. However, there are (at least) two ways in which we can eliminate some of the suggested origins of the 260 day period based upon either latitude or cultural isolation. For this reason we have investigated a site (Stonehenge in Wiltshire, UK) remote in distance (8876km of ocean) and time (2600 years earlier based on a date of AD 1 [WINTER95] for Building J) from Mesoamerica for which find similar, but much stronger, evidence for the architectural use of the 365/260 ratio. We propose an astronomical origin for 13×20=260.

But first, we re-examine the ratios found by Peeler and Winter at Monte Albán.

2. The Ball Courts at Monte Albán

Peeler and Winter reports careful measurements of the lengths of the two excavated ball courts at Monte Albán (correcting a previous error) and noted that the ratio (1.401447) of the two lengths, 40.67m and 29.02m was close to the ratio (1.403846) of the solar (365) and ritual (260) calendars. They proposed that this was not a chance coincidence, but was evidence of human design. This proposal was supported by other evidence at Building J (Section 3, "*Building J, the 'Observatory', at Monte Albán*") and at Tlailotlacan at Teotihuacan (Section 5, "*Tlailotlacan at Teotihuacan*"), but for the moment we restrict out attention to the ball courts alone. Peeler and Winter stress that they leave open the question of any recognition of a non-integral number of days in the solar year (365.24218408) and concentrate on the ratio of integral values. This seems eminently very reasonable. Although it is obvious that the Mesoamericans could count, even to very high numbers, it is very unlikely that they had any concept of non-integral values corresponding to our decimal system.[ii] In this section we investigate the question of whether the ratio of ball court lengths can be identified with any ratio of two integral values. This will involve a question of approximation–how close does the integral ratio have to be to the measured ratio to be acceptable. We introduce a percentage error to define an acceptable identity, and tabulate these identities for a range of errors. As we see in Table 1, "Monte Albán, Ball Court Ratios" the measured ratio can be identified with many integral ratios. There are other integral ratios with errors greater than those found at Mont Albán, but we halt our search as soon as we find the ratio 365/260.

Table 1. Monte Albán, Ball Court Ratios

delta (δ)	Number of ratios found	Numerator	Denominator	Error (%)
0.00001	0			
0.0000126	1	192	137	-0.000897
0.00002	2	391	279	-0.000969
0.00004	4	199	142	-0.002770
		377	269	0.002834
0.00007	6	405	289	+0.004509
		185	132	+0.004843
0.00009	7	206	147	-0.006189
0.00010	8	363	259	+0.006930
0.00015	11	419	299	+0.007813

[ii] It would seem likely that they would be aware of the concept of a *half* and possibly a *half of a half*, but these concepts do not appear to have been included in any calendrical accounts.

		178	127	+0.009100
		213	152	-0.009382
0.00020	15	349	249	+0.11356
		220	157	-0.012372
		171	122	+0.013705
		447	319	-0.013797
0.00025	18	227	162	-0.015178
		335	239	+0.016152
		234	167	-0.017815
0.00030	21	164	117	+0.018704
		321	229	+0.021367
		241	172	+0.020300
0.00035	24	248	177	-0.022643
		157	112	+0.024149
		255	182	-0.024858
0.00040	27	307	219	0.027058
0.00050	35	436	311	0.034392
0.00100	74	115	82	0.070766
0.00150	91	7	5	0.103270
		383	273	0.105828
0.00200	108	327	233	0.141725
0.00230	134	333	238	-0.163232
0.0023988761	142	298	213	-0.170270
0.0023988762	143	73 (365)	52 (260)	0.171171

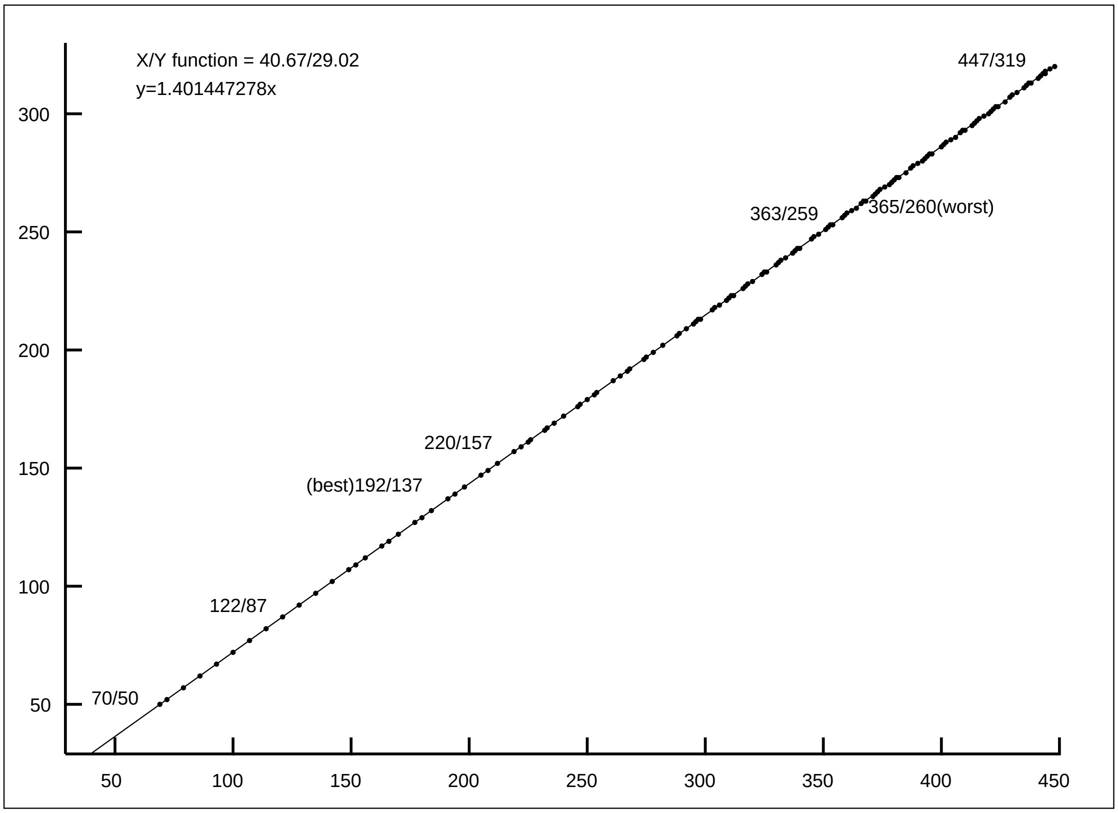
In this table we are looking for ratios of integral numbers that approximate to 40.67/29.02=1.401447278. In the first column, we set a percentage error that we regard as the upper acceptable limit to identifying a ratio. In the second column we list the total number of identities found at the error level in column 1. In the third and fourth columns are the integers found, and in the fifth column the percentage accuracy achieved by those integers. We have restricted out investigation to those ratios whose error is equal to or less than that for the 73/52=365/260 ratio (0.171171%), and also exclude any ratios which are simple multiples (e.g. 365/260 is rejected in favor of 73/52), and any with a with a numerator greater than 450. In total we find 143 acceptable integer ratios with errors less than or equal to 365/260. The table omits many ratios whose error limits are 0.0040% and larger, but all are included in Figure 1, "Ratios of Integrals close to 40.67/29.02". Firstly we note that if we insist on an error limit less than 0.0000126% (1 part in 8 million) then the measured ratio cannot be identified with any integral ratio; surely it would be unreasonable to expect the Zapotecs at Monte Albán to have measured lengths to this accuracy. Secondly we note that the measured ratio approximates to 142 different integral ratios which are all at higher accuracy then for 365/260. There is nothing in the measured ratio itself to encourage identification with 365/260; 192/137[iii] is a much better choice, and is almost 200 times better than 365/260. Nevertheless, all these 143 pairs of integers do exhibit a ratio acceptably close to 40.67/29.02.

There are of course an infinite number of ratios exactly equal to 40.67/29.02=1.401447278, given by the linear function y=1.401447278x. The number of these that we can regard as ratios of integers is limited by the error margin we are prepared accept. The exact identity between the values is represented by the full line in Figure 1, "Ratios of Integrals close to 40.67/29.02", and ratios of integrals are marked as points on the line. At this scale of representation, all points appear to be on the line. All the 143 points found are plotted, and a few of the ratios are marked. If we now decide that the ratio of the lengths of the ball courts were designed to represent a ratio of integral values, then we have to choose one of the points in this figure. Which one? It might seem most appropriate to choose the one closest to the line: 192/137. (Or 122/87 which appears in all three sites studied by Peeler and Winter, see Table 6, "The Ratios Common to the Ball Courts and Building J at Monte Albán, and Tlailotlacan at Teotihuacan") but unfortunately these numbers have no other significance for us (as yet). Peeler and Winter in fact chose the worst fit, 365/260, because it had meaning in terms of the calendars in use by the Mesoamericans.[iv]

[iii] I attach no significance at all to the identity of the 137 with the dimensionless fine structure constant 1/137 (actually 1/137.036) introduced by Sommerfeld in 1916, which occurs in cosmology and quantum mechanics and determines the spectra of light from the sun.

[iv] The process reminds me of the mistaken attempts to show that the Golden Section, (√5+1)/2=1.618033989, was used in the construction of the pyramids in Egypt and in many paintings; the Golden Section had meaning outside the measurements.

Figure 1. Ratios of Integrals close to 40.67/29.02



Considering the lengths of the ball courts in isolation, we should accept that if integral ratios were important to the builders, the construction went according to the ratio 192/137=1.401459854 as closest to 40.67/29.02=1.401447278, and we must assign the ratio 365/260=1.403846154 to not only a coincidence, but a rather poor coincidence.

The only reason for preferring 365/260 is that these numbers are familiar as the (approximate) number of days in the year, and the length of the ritual calendar, so the best ratio found was rejected and the poorest accepted as meaningful. This is normal science: one explanation covering two independent observations is better than two different explanations.

We can now test the hypothesis that the ratio 365/260 was a deliberate intention in Zapotec structures by examining other buildings and sites. In the next sections we apply the same analysis to the other instances of identification with 365/260 noted by Peeler and Winter: Building J at

Monte Albán, and Tlailotlacan at Teotihuacan. We expect some of the ratios to occur at all sites, and that 365/260 would appear higher in the list of common ratios if the proposal of Peeler and Winter is to be upheld. As a final test we will search for the same 365/260 ratio at a site remote in distance and time from Mesoamerica where the ritual 260 day calendar was presumably unknown (Stonehenge).

3. Building J, the 'Observatory', at Monte Albán

The same analysis for the re-constructed base plan for Building J at Monte Albán is presented in Table 1, “Monte Albán, Ball Court Ratios”. The results are very similar to those for the ball courts: no identity with an integral ratio with a percentage error less than 0.00125854 (1 part in 5.6 million), and 70 different ratios with errors less than that for Building J. The error for 365/260 (0.07742%) is certainly less than that for the ball courts (0.17717%), but we note that the measurements for Building J are based on a re-constructed hypothetical base plan for which no physical evidences exists on the ground, and that the best integral ratio has an error 61.5 times less that for 365/260. Again, in isolation, we see the identity of the ratio of lengths with 365/260 is most likely not only coincidence, but a poor coincidence. However, although many of the ratios found also occurred in the ball courts, the existence of the 365/260 ratio does add further support to an identification with the calendars.

Table 2. Building J, the Observatory, at Monte Albán

delta (δ)	Number of ratios found	Numerator	Denominator	Error (%)
0.0000176542	0			
0.0000176543	1	101	72	+0.00125854
0.00003	2	411	293	-0.00212068
0.00005	3	310	221	-0.00322160
0.00007	4	397	283	+0.00475717
0.00008	5	209	149	-0.00538650
0.00009	6	296	211	+0.00595101
0.00011	7	317	226	-0.00750351
0.00012	9	195	139	+0.00838164
		425	303	-0.00854454
0.00016	11	289	206	+0.01087127
0.00017	12	108	77	-0.01160005
0.00018	13	383	273	+0.01213889
0.00021	14	439	313	-0.01455794
0.00022	15	331	236	-0.01552301
0.00023	16	94	67	+0.01603632

0.00025	17	223	159	-0.01742281
0.00028	18	338	241	-0.01928319
0.00029	19	369	263	+0.02008195
0.00031	20	275	196	+0.02146490
0.00033	21	115	82	-0.02289052
0.00035	22	181	129	+0.02428439
0.0010860303	70	164	117	-0.07490388
0.0010860304	71	73 (365)	52 (260)	+0.07742096

Before we examine the evidence for the 365/260 ratio at Teotihuacan, we consider first the only significant solar alignment found at Monte Albán, the 108° azimuth of the nadir solar passage. This alignment points to Building O at Caballito Blanco in the Tlacolula valley some 35km from Monte Albán.

4. Building O at Caballito Blanco

The relationship between Building J at Monte Albán and Building O at Caballito Blanco is crucial to the argument of Peeler and Winter, it appears to be the sole supporting evidence that the Zapotecs at Monte Albán had any interest in the zenith and nadir passage events and the corresponding angle of 36°. The evidence connecting this alignment with the 365/260 ratio in Building J and the ball courts is rather tenuous: Peeler and Winter claim that the alignment from Building J to the rising of Capella on the day of the solar zenith is "*precisely parallel*" to the line joining the centers of the ball courts (whose lengths are approximately in the ratio 365/260). We examine this important relationship in some detail.

Peeler and Winter noted that the line joining Building J and Building O is close to the 108° azimuth of the nadir passage of the sun on August 5, and believed it close enough to indicate deliberate design. The coordinates of the two buildings are given in Table 3, "Buildings J at Monte Albán and O at Caballito Blanco". From this we find the distance between J and O to be 35.00km (in agreement with the 36km recorded by Peeler and Winter), and the orientation 107°46'05.44" (in agreement with the 108° recorded by Peeler and Winter). The difference of 14' in the orientation corresponds to a point only 150m north of Building O, a very acceptable error over a 35km distance. The identification of the J-O line with the azimuth of the summer nadir passage sunrise is well established. However the direction of the nadir sunrise in the east at 108° can only be indirectly observed by the sunset in the west at 288° on the day of the zenith, and this line of sight from Building J is obscured by Building M. A speculative and convoluted construction sequence might be: first establish the 288° line from J (in spite of the obstruction by M), then project this backwards along 108° to some point on the Cerro Yani Grande ridge, then from this ridge project the line further over the Tlacolula valley, then search along that line for a convenient site for Building O, finally locating Caballito Blanco. to complete this sequence

Table 3. Buildings J at Monte Albán and O at Caballito Blanco

Building	Coordinates	Decimal Degrees	Grid Coordinates
J (Monte Albán)	17° 2' 38.360" N	17.0439888° N	37600,85300
	96° 46' 2.992" W	-96.7674673° W	
O (Caballito Blanco)	16° 56' 48.432" N	16.9463198° N	72200,75400
	96° 27' 15.696" W	-96.4542792° W	

There are (at least) six points that suggest that the location of Building O might not be the result of deliberate design, but even cumulatively they cannot be said to rule out deliberate design.

- Although the arrow-shaped layout of Buildings J and O are very similar, the orientations of their major axes differ by almost 30°. The orientation at Building J has been associated with the rising of Capella at the time of the zenith passage of the sun ([WINTER95] and previous references therein). One might have expected the same orientation at Caballito Blanco if the association between them had astronomical content.

- The ratio of the sides of the base triangle constructed by Peeler and Winter for Building J (77.25/55.07=1.40276) is very close to 365/260=1.40385, A construction similar to that used by Peeler and Winter for Building_J produced a re-constructed base plan for Building O at Caballito Blanco. In this case the resulting triangle is very nearly isosceles and the ratio of side lengths is very close to 1.500, a long way from 365/260.[v]

- The indirect observation of the nadir passage depends upon a 288° sight line from Building J, but this is blocked by Building M. Similarly the 108° line of sight from Building J is blocked by Building Q. It is almost as if Building J is located between high walls in the 108° and 288° directions. Could the height of Building J have been sufficient to allow oversights of Buildings M and Q? Or could Building J have preceded Buildings M and Q?

- As noted earlier, the line of sight between Buildings J and O is also obscured by the ridge and peak of Cerro Yari Grande.

- There is no obvious sight line within or from Monte Albán to the direct observation of the zenith sunrise at 72°, needed along with the nadir at 108° to define the 36° angle

- Peeler and Winter relate Building J to the 2 ball courts because a line joining the centers of the ball courts is "*precisely parallel*" to the 47°57' perpendicular line from the base of the J stairs that passed over Building P to the rising of Capella in AD 1 on the day of the solar zenith.

  The coordinates of the centers of the ball courts are: Large: 17°02'38.16" N and 96°46'02.03" W and Small: 17°02'48.36" N and 96°45'51.76" W. This a separation distance between centers of 437.3m and an azimuth of 43°54'35". This is to be compared with compared this the 47°57'recorded by Peeler and Winter for the Capella rising–a 4° difference which corresponds to about 30.6m difference in the location of the 29.02m small ball court. The point 17°02'48.35" N,96°45'52.82" W, derived from the azimuth of the Capella rising, is 30.7m from center of small ball court. This is within the coach park.[vi] A 4° difference between these line is hardly "*precisely parallel*", and the link between the 108° O-J line and the ball courts is indeed tenuous.

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[v]We have not measured Building O, and lengths in mm were taken from [WINTER95] as 25.5/17.0, exactly 1.5, so we get a perfect fit to a ratio of 3/2. If these (poor) estimates of lengths were in error by 0.5mm, the extremes for 365/260 are: -10.910% for 26/16.5, -6.410% for 25.5/17.0, and -1.731% for 25/17.5. To get the ratio 365/260 would require a sides ratio of 24.82/17.68 which is far outside the measurement errors. If there was a deliberate design plan for the two buildings, one would have expected a similar base plan.

[vi]We obtained coordinates from Google Earth, and to get an idea of the reliability of these, we measured the length of the large ball court from 17°02'38 85" N,96°46'01.93" W to 17°02'37.58" N, 96°46'02.10" W, which gives a length of 39.55m (Peeler and Winter measured 40.67m on the ground), with an azimuth of 187°17'35"

- The orientations of the ball courts, 187°17' and 272°31' (see footnote) bear no relation to the zenith and nadir passages of the sun st 72° and 108°.

If we were to accept the points listed above, then we would be left with only a single point indicating any Zapotec interest in the solar passages, the very close alignment of Buildings J and O with the zenith passage of the sun. We now proceed to consider the evidence at Tlailotlacan.

Similarly, for the small ball court from 17°02'48.36" N,96°45'51.27" W, to 17°02'48.40" N,96°45'52.22" W gives a length of 28.08m and an azimuth of 272°31'18" (Peeler and Winter measured 29.02 on the ground).

Given the uncertainties of locating the ends of the ball courts on enlarged satellite photographs, we fee that the Google Earth figures look reasonably reliable.

5. Tlailotlacan at Teotihuacan

5.1. The Ratio at Tlailotlacan defined by Lines of sight over the Pyramids of Quetzalcoatl and the Moon

The measurement of the distance between Tlailotlacan and the pyramid of Quetzalcoatl (the 365 element) is not a measurement between two locations clearly identified on the ground, but between a point in Tlailotlacan identified by assuming an angle of 36° between the lines of sight over the pyramids of Quetzalcoatl and the Moon, and the line over Quetzalcoatl being approximately 108°. The distance between Quetzalcoatl and the Moon (the 260 element) is of course between two large pre-existing structures.

In Table 4, "Ratios Between Tlailotlacan and the Pyramids of Quetzalcoatl and the Moon" we repeat the ratio analysis for the lengths recorded by Peeler and Winter. We see the same pattern emerging, no integral identity below an error of 0.00020359%, 50 identities at higher accuracy than 365/260, and the best identity (275/196) some 274 times better than 365/260.

Table 4. Ratios Between Tlailotlacan and the Pyramids of Quetzalcoatl and the Moon

delta (δ)	Number of ratios found	Numerator	Denominator	Error (%)
0.0000028565	0			
0.0000028566	1	275	196	-0.00020359
0.000023	2	369	263	-0.00158624
0.000037	3	181	129	+0.00261529
0.000061	4	449	320	+0.00434185
0.000078	5	94	67	-0.00563099
0.000080	6	268	191	+0.00550796
0.000098	7	355	253	+0.00698288
0.000111	8	442	315	+0.00787720
0.000135	9	383	273	-0.00952758
0.000152	10	289	206	-0.01079492
0.000200	12	195	139	-0.01328401
0.000300	20	167	119	+0.02118674
0.000400	27	407	290	+0.02738256

0.000500	31	108	77	-0.03326138
0.000600	39	338	241	-0.04094285
0.000700	46	237	169	-0.04969246
0.0007820728	50	122	87	-0.05453995
0.0007820729	51	73 (365)	52 (260)	+0.05574035

Again, in isolation, we see the identity of the ratio of lengths with 365/260 is most likely not only coincidence, but a poor coincidence. Just how much of a coincidence we investigated by a simulation. First we chose two random integers, one in the range 1-450, the other in the range 1-320, calculated the ratio of the larger over the smaller, and checked if it came within the range 365/2600.005. This was repeated a billion times and we found 3582579 hits, a success rate of 0.358%. The chance of finding the ratio 365/2600.005 by chance alone out of all possible ratios of 1-450/1-320 is about 1 in 185, unlikely but certainly not impossible odds. However this treatment assigns equal likelihood to inappropriate ratios such as 365/1, 365/2, 450/3 etc. Restricting the range searched to something more reasonable yielded much higher likelihoods of finding 365/260. Searching the range of ratios between 1.35 and 1.45 (a range of 0.1 symmetrically around 1.40) yielded a 17.275% chance of hitting 365/2600.005, odds of 5.9 to one, essentially the same as throwing a six with a die. This is obviously a very realistic probability of getting 365/260 by chance alone, but it is not clear just what range of ratios to search. As the ratio range is extended the likelihood drops of course, see Table 5, "Dependence of Finding a Ratio of 365/260 upon Range of Ratios".

Table 5. Dependence of Finding a Ratio of 365/260 upon Range of Ratios

Search Range	Ratio: From-To	Numerator range	Denominator range	Percent found	Odds
0.1	1.35 - 1.45	360 - 370	256 - 264	17.275	5.798
0.2	1.3 - 1.5	355 - 375	248 - 270	8.397	11.909
0.3	1.25 - 1.55	350 - 380	249 - 271	5.548	18.025
0.4	1.2 - 1.6	345 - 385	236 - 280	4.201	23.804
0.6	1.1 - 1.7	336 - 394	222.289	2.741	36.483
0.8	1.0 - 1.8	328 - 402	208 - 297	2.082	48.031
1.0	0.9 - 1.9	321 - 409	191 - 304	1.644	60.827

The odds against finding even the 17% chance event three times (ball courts, Building J, and Teotihuacan) reduces the chance to 0.516%, about 194 to 1 which is an unlikely but not impossible occurrence. This of course relies totally upon the independence and validity of the three events, and only one case (the ball courts) is a direct measurement between clear-cut features on the ground. The constructions required at Teotihuacan and Building J are both definitely anticipatory of the 365/260 ratio, casting some reservations upon the threefold coincidence, and we feel that a verdict of *not-proven* is the best we can do.

We now look more closely at the geometry of the construction at Tlailotlacan. Combining the requirements of an azimuth of approximately 108°, and sight lines over Quetzalcoatl and the Moon of 36°, Peeler and Winter show that the geometry of Tlailotlacan and the two pyramids is a unique and fixed triangle. We use the geometry in Section 5.2, "*The geometry at Tlailotlacan*".

5.2. The geometry at Tlailotlacan

The technique employed by Peeler and Winter is described explicitly in [WINTER10] page 15. First they located a point to the west of the Avenue of the Dead and about 400m to the east of Tlailotlacan where the zenith and nadir passages occurred directly over the pyramids of Quetzalcoatl and the Moon[vii] but found no significant marker on the ground. They then found that if they moved westwards into Tlailotlacan, they could locate a point at which the angle between sight lines over the pyramids was exactly 36° (the angle at Monte Albán) but of course the sight lines over the pyramids no longer marked the zenith and nadir passages. However, at this location in Tlailotlacan they found the ratio of two sides of the triangle approximated to the ratio 365/260 with the error quoted above.

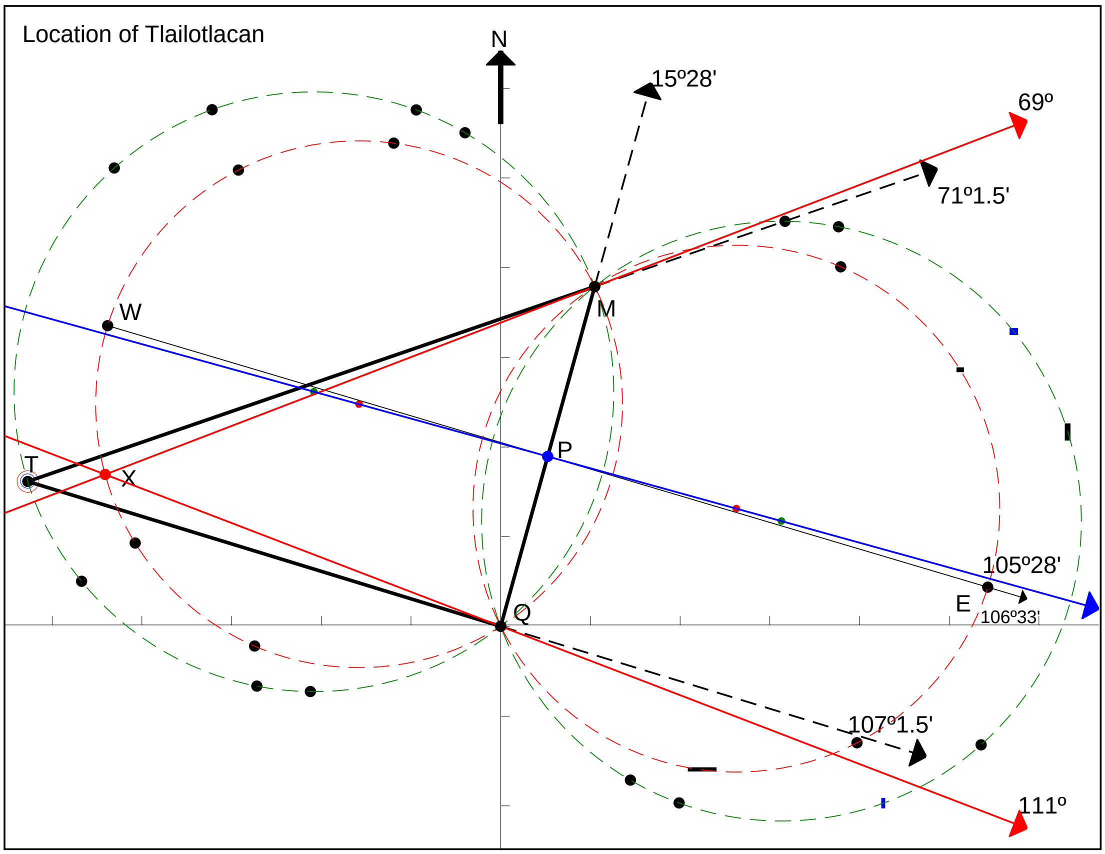
We now investigate systematically. Firstly, the location *T* in Figure 2, "Locus of Possible Locations of Tlailotlacan" must subtend an angle of 36° between the lines of sight over the pyramids of Quetzalcoatl *Q* and the Moon *M*. The nomenclature refers to the geometry displayed in Figure 2, "Locus of Possible Locations of Tlailotlacan". The locus of these points is the outer circles in the figure, and there are an infinite number of points satisfying this condition. Secondly, if the ratio of any two sides[viii] must be equal to 365/260, the possible locations for *T* are limited to the 16 points marked on the outer circles. All of these points satisfy the conditions of a 36° angle and a ratio of 365/260. One of them, of course, is very close to the point in Tlailotlacan identified by Peeler and Winter, but we stress that *none* of these points generates sight lines to the zenith and nadir passages at Teotihuacan. If we now require the lines of sight over the pyramids to point eastwards and roughly, but inexactly, towards the passages of the sun, we arrive at a unique location *T* which is very close to that selected by Peeler and Winter. (The difference in location between this point and that recorded by Peeler and Winter is too small to be distinguished on Figure 2, "Locus of Possible Locations of Tlailotlacan", but is responsible for the error in the 365/260 ratio. For the calculated location of *T* we find the *TQ* vector has an azimuth of 107°1.5', near enough to the 108° nadir azimuth at Monte Albán, but far from the 111° passage at Teotihuacan. The ratio *TQ/QM* is

[vii]They actually investigated all possible combinations of the pyramids of Quetzalcoatl, Sun, and Moon, east and west of the Street of the Dead, but found no significant markers on the ground for any combination.

[viii]we relax the condition that the ratio must be defined by *TQ/TM* to allow any pair of the distances *TQ*, *TM*, and *QM* to define the ratio 365/260. We see no reason to restrict the ratio to *TQ/TM.*

exactly 365/260. This is in complete agreement with Peeler and Winter apart from a very minor change in location of *T*.

Figure 2. Locus of Possible Locations of Tlailotlacan



In this figure (Figure 2, "Locus of Possible Locations of Tlailotlacan") *T* marks the location in Tlailotlacan identified by us and by Peeler and Winter. *M* and *Q* are the locations of the Pyramids of the Moon and Quetzalcoatl respectively. Point *P* is the mid point of *QM*, and the longer arrowed line through *P* is the direction 105º28' to the sunset on August 12/13, a direction which celebrates the beginning of time and also enshrines the 260 day count. The outer circles are the locus of an infinite number of points subtending an angle of 36º to *M* and *Q*. The points marked on these circles are all centered on locations where the where the *MTQ* angle is exactly 36º, and the ratio of one pair of the two sides of the triangle is exactly the ratio 365/260. The analysis for the correct angle (42°) between the lines of sight over the pyramids is marked in Figure 2, "Locus of Possible Locations of Tlailotlacan" on the inner circles. The unique solution (*X*) is approximately half a kilometer east of Tlailotlacan has the correct lines of sight, but is 3° (6 sun-widths) away from the 108° line and is far from the 365/260 ratio.

The problem we see with this solution is that the lines of sight over the pyramids are far from the zenith and nadir passages of the sun, 71°1.5' and 107°1.5' as opposed to the 69° and 111° at the latitude of Teotihuacan (4 and 8 sun-widths out respectively). Peeler and Winter are well aware that the lines of sight are not correct, but stress that they would be correct at Monte Albán, implying that the angle of 36° was of greater significance to the Zapotecs than lines of sight corresponding to important solar events. We find this rather unrealistic, the activity of seeing the sun appear in the right place at the right time has far more impact than knowing that the angle between the rising sun at zenith and nadir is 36° in Monte Albán.[ix]

Accepting the pre-determined inclination of the Street of the Dead to geographic north (15°28' east of north) and the distance (1964.7m) between the pyramids of Quetzalcoatl and the Moon Peeler and Winter propose that the site of Tlailotlacan was deliberately chosen to reproduce the ratio 365/260 and the angle of 36°. This proposal, however, leaves us with a difficulty. Was the location of Tlailotlacan chosen by the Zapotecs to mark the sight lines of the zenith passages of the sun (69° and 111°, a difference of 42°) over the pyramids of Quetzalcoatl and the Moon. Or was the intention to enshrine at Teotihuacan the angle between the same events at Monte Albán (72° and 108°, a difference of 36°). Peeler and Winter proposed that the intention was to enshrine the angle of 36°, even though that resulted in sight lines of the zenith events that were seriously in error. If, on the other hand, the intention was to mark the sight lines of the zenith events, we must accept that the Zapotecs simply *got it rather badly wrong*, the site of Tlailotlacan should have been about half a kilometre to the east. Either way it appears that the Zapotecs at Teotihuacan were unaware of the relationship between the angle of 36° and the sight lines of zenith events. It is also significant that there is no evidence that both these sight lines were marked out in the architecture at Monte Albaán.

We stress that at NO point on the outer circles are the sunrises of the zenith and nadir passages of the sun observed over *M* and *Q*, these events only occur on the inner circles, and only at the point *X*. Two other points were made by Peeler and Winter as helping to determine the location of Tlailotlacan. Firstly, the sight line from *T* over *Q* points to the August 12/13 sunset at 105°28'. In fact the line of sight is 107°1.5' an error clearly visible as three sun diameters. Secondly they claim the angle *TQM* is exactly 90° in support of the August 13 line. In fact it is 88.4°–again the three sun diameter error.

We note in passing that the points *W* and *E* define a direction which is exactly at right angles to *QM* (106°33') which happens to be close to the August 12/13 direction (105°28'); we doubt that the line *WE* is of any significance although it does emphasise the angle 42°.

To give some idea of the sensitivity of the ratio at these locations, rectangles corresponding to an error range ±1% in the 365/260 ratio are given for one point on each of the circles: the short

[ix]I am reminded of once watching the sunrise at Stonehenge close to the solstice, and seeing the sun perched exactly on the point of the Heel Stone. The fact that this was exactly at an angle of 51.3° east of north did not register.

black lines on the eastern circles. Varying the angle at Tlailotlacan by ±1% makes very little difference to the location, see the blue rectangles. We conclude that the size of the points marked on the circles are a very fair representation of likely error margins. The point *T* is actually a superposition of two locations: that given by Peeler and Winter which differs from the 365/260 ratio by 0.0557%, with *TQ* and *TM* vectors of 107º1.48' and 71º1.48', and that produced by this program, 2.44m to the east of the Peeler and Winter location, differing by only 0.000001%, with vectors 107º4.00' and 71º4.00'. These points cannot be distinguished on the scale of the figure, but are indicated by the apparently concentric circles around the point.

In summary we find:

- There are an infinite number of locations (the inner circles) where the angle between the sight lines over *M* and *Q* is exactly 36°.
- There are an infinite number of locations (the inner circles) where the angle between the sight lines over M and Q is exactly 42°.
- There are 16 locations where the angle between the sight lines is exactly 36° and the exact ratio 365/260 is found.
- There are 10 locations where the angle between the sight lines is exactly 42° and the exact ratio 365/260 is found.
- There is only one unique location (*X*) at which the sight lines over the pyramids of Quetzalcoatl and the Moon correspond to the zenith passages of the sun with an angle of 42°, but the ratio of *XQ/QM*=1.2019 is far from 365/260=1.4038. And *X* is half a kilometer distant from *T* in Tlailotlacan.
- There is only one unique location (*T*) at which generates an angle of 36°, and where the ratio of *TQ/QM*=1.40381 is very close to 365/260=1.40384. However, the sight lines over the pyramids of Quetzalcoatl and the Moon from *T* do not correspond to the zenith passages of the sun.
- There is NO location at Teotihuacan where the sight lines to the solar passages at Teotihuacan (69° and 111°) are correct, and the ratio is 365/260.
- There are two points (*W* and *E* on Figure 2, "Locus of Possible Locations of Tlailotlacan") passing through the mid-point, *P*, of *QM* which define an azimuth, 106°22' very close to the perpendicular to the Street of the Dead (105°27'). These points also define the 42° angle and the ratio 365/260, but do not provide lines of sight over *Q* and *M* to the solar passages.
- As Peeler and Winter noted there is only one unique point which simultaneously, but only approximately, satisfies the conditions of the 36° angle, the 365/260 ratio, and the TQ sight line (107°1.5') close to either the 108° nadir passage of the sun at Monte Albán, or the perpendicular to the Street of the Dead (105°28').

In view of these findings it is perhaps tempting to regard the location of Tlailotlacan as coincidentally close to the angle between solar zenith observations at Monte Albán and the ratio

TQ/QM approximately 365/260. However, in view of the observations at Monte Albán and the very well documented cultural significance of the 260 period as the ritual calendar in Mesoamerica, perhaps we should explore further.

5.3. Combined Observations at Monte Albán and Teotihuacan

We now consider those ratios which are identified at all three sites. In our analysis we have ensured that 365/260 occurs in all three, but we find that 50 different integral ratios also occur at all three sites, and that all 50 are closer to the measured ratio than 365/260. These ratios are listed together with their errors in Table 6, "The Ratios Common to the Ball Courts and Building J at Monte Albán, and Tlailotlacan at Teotihuacan"

Table 6. The Ratios Common to the Ball Courts and Building J at Monte Albán, and Tlailotlacan at Teotihuacan

Numerator	Denominator	Error (%)
445	317	0.05825142
122	87	0.06076382
359	256	0.06396761
237	169	0.06561689
352	251	0.06729904
115	82	0.07076589
338	241	0.07437660
223	159	0.07623872
331	236	0.07814030
108	77	0.08206694
317	226	0.08616732
209	149	0.08828631
310	221	0.09045323
101	72	0.09493757
296	211	0.09963444
195	139	0.10206735
289	206	0.10455931
94	67	0.10972920
369	263	0.11377862

275	196	0.11516286
181	129	0.11798499
449	320	0.11971355
268	191	0.12088100
442	315	0.12325297
87	62	0.12690657
428	305	0.13067996
341	243	0.13164272
254	181	0.13326505
421	300	0.13457913
167	119	0.13657787
414	295	0.13861048
247	176	0.13998480
327	233	0.14172482
407	290	0.14278083
80	57	0.14709752
439	313	0.14886435
425	303	0.15036911
393	280	0.15156837
411	293	0.15197658
313	223	0.15271115
397	283	0.15369765
233	166	0.15463872
383	273	0.15554481
386	275	0.15620180
153	109	0.15858228
355	253	0.15967719
379	270	0.16100684
226	161	0.16264831
299	213	0.16472905
372	265	0.16599320
73	52	0.17117134

With 50 different ratios common to all three sites, we see the identity of the ratio of lengths with 365/260 is most likely not only coincidence, but a poor coincidence. We have not attempted to identify any of the other ratios common to all three sites with any non-architectural feature. We now examine the possibility of the ratio 365/260 occurring at other sites.

5.4. Other possible sites exhibiting the 365/260 ratio

The scope for examining other Mesoamerican sites is almost limitless, but our experience with the three sites above suggests that any search could well produce the same result. However, there is one possibility that could lead to significant results.

We can test the relevance of the 365/260 ratio by examining the geometry of a site far removed from Teotihuacan in both time and distance, a site where there has been no evidence, documentary or otherwise, of a 260 day ritual calendar, and where there is no possibility of cultural interaction with Mesoamaerica. We select the site Stonehenge I (now known as Stonehenge 3 I) in southwest England as sufficiently removed in time (at least 2500 years earlier) and distance (almost 9000km) to eliminate any possibility of cultural contact with the inhabitants of Mesoamerica. Furthermore, there is no question of zenith and nadir passages of the sun at the latitude of Stonehenge where the sun is never directly overhead. If we find evidence for the ratio 365/260 at Stonehenge, that would seem to rule out coincidence, and would also indicate a universal, world-wide significance of the 260 day period.

6. Stonehenge

6.1. Description

The earliest substantiated structure at Stonehenge, now known as Stonehenge 1, which has been dated to about 3100 BCE, was a circular bank and ditch about 110m in diameter, with a wide entrance to the north east, approximately oriented towards sunrise at the summer solstice, and with a narrower entrance on the opposite, south west, side.[x] Just within this ditch an almost perfect circle of the 56 equally spaced holes were dug, now known as the Aubrey Holes.[xi] In the period labelled Stonehenge 2, ca. 3000 BCE, more post holes appear to indicate a possible wooden structure within the circle, and a line of post holes from the south west entrance follow the line to the center of the circle. The next phase of construction, known as Stonehenge 3 I, ca. 2600BCE, is the one we examine in this paper.[xii]

This period, Stonehenge 3 I, ca. 2600 BCE, included a rectangle marked by a standing stone at each corner. The two shorter sides point closely to the midsummer sunrise in the period around 2500 BCE, and a lone stone (known as the Heel Stone, numbered 96) lying on an extension of the bisector of the rectangle points in the same direction. The four stones of the rectangle are known as the Station Stones, and are numbered 91-94. At midsummer the solstice sun rises along 92-91 and 93-94 and over the Heel Stone as viewed from the center of the rectangle, and the two summer full moons rise along either 93-92 and 94-91 or the diagonal 93-91. At midwinter the directions are reversed and the solstice sun sets along 91-92 and 94-93, and the midwinter full moons set along 91-94 and 92-93 or 91-93. The Station Stones lie very closely on the almost perfect circle of the 56 Aubrey Holes but it is clear which came first as the mound and ditch surrounding stone 92 are super-imposed on Aubrey holes 17, 18, and 19. These five stones we take as the primary stone structure Figure 5, “The Lengths of the Rectangle and the Distance of the Heel Stone”, but we also include four major points along the primary axis; the intersection, *T*, of the main axis with the southern arc of the Aubrey circles (an important point defined earlier in Stonehenge 2.), the mid points of 91-94 (*X*), and 92-93 (*Y*), and the center, *C*, of the both the Aubrey circle and the Station Stones. This structure of Stonehenge 3 I clearly long pre-dated the other circles (such as the Y and Z holes), and the great trilithons and bluestone circle and horseshoe of Stonehenge 3 II, 3 IV, and 3 V.[xiii] The geometry of this original structure is given in Figure 5, “The Lengths of the Rectangle and the Distance of the Heel Stone” below using the positions recorded on the plan issued by the Ministry of Public Building and Works in 1959 [NEWALL65]. Lines of sight between the Heel and Station Stones were determined with greater accuracy by Hawkins [HAWKINS63] and later expanded by him [HAWKINS65].

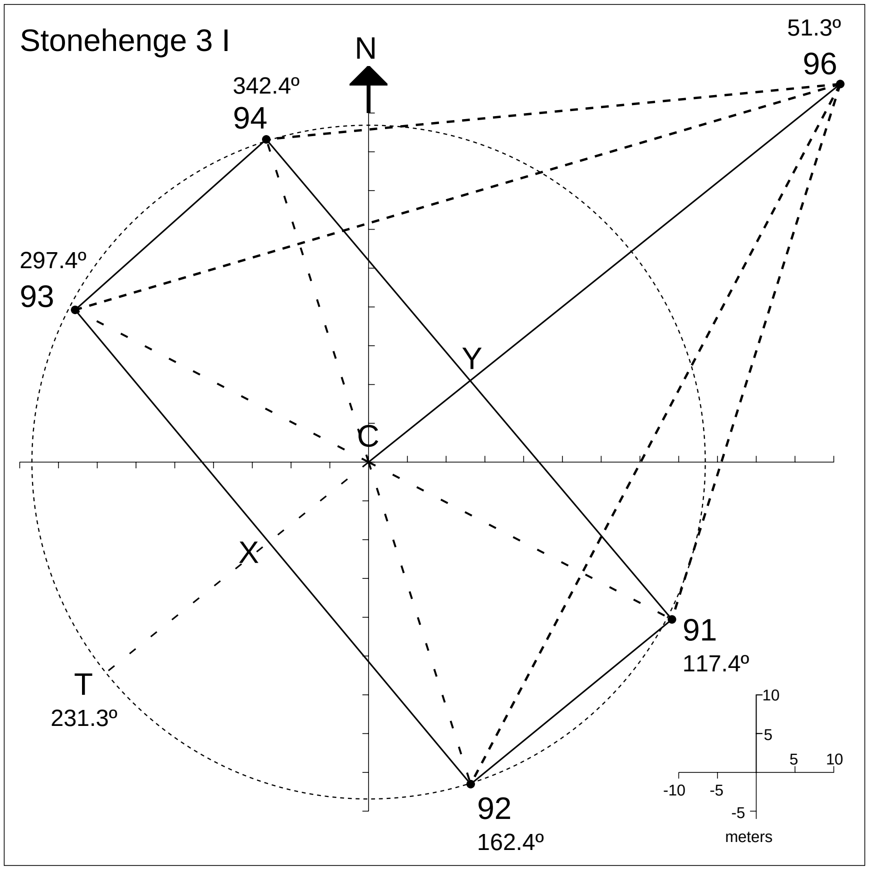
[x] It appears probable that even earlier post-holes dating back to perhaps 8000 BCE had held pine posts.

[xi] These holes were apparently dug and re-filled almost immediately with white chalk. Many of them were re-opened later to receive inhumations.

[xii] The strange nomenclature serves to include an older definition of periods when this was labelled simply Stonehenge I. The older nomenclature is often used in the work referenced in this paper

[xiii] the original Stonehenge III has now disappeared and is subsumed in IV and V.

Figure 3. Basic Geometry of Stonehenge 3 I



6.2. Investigation, ratios of lengths

The locations of the stones and the sight lines they generate have long been associated with significant observations of the setting and rising of the sun and moon at critical times of the year. We have now examined, for the first time, the ratios of the distances between these stones, (96-92)/(96-91)=1.4127, (96-93)/(96-94)=1.4000, (96-X)/(91-93)=1.3983, and (96-X)/(92-94)=1.4124 and find that they are all close to the ratio (1.403846154) of the length of the solar year (365 days) and the (Mesoamerican) ritual year (260 days). With a mean of 1.4059 (0.15% from 365/260) and a standard deviation of 0.00694 (0.49%) it appears at least probable that the ratio 365/260 determined the basic structure of Stonehenge 3 I, some 2500 years earlier and 9 000 km distant from Teotihuacan. Clearly there was no possibility of cultural contact between these peoples, and we are left with a choice of either coincidence or a significant common determinant. However, it is clear that whatever the origin of the 260 day period, it was independent of latitude, and was not confined to Mesoamerica.

A complete analysis of Stonehenge geometry, which included the center of the structure, and the mid-points of 91-94 and 92-93 and the intersection of the main axis with the circle of Aubrey holes opposite to the Heel Stone involved 9 points which generated 36 different lines, with a total of 630 pairwise ratios[xiv] which were examined for closeness to the ratio 365/260.

The probability of finding *x* cases of identity with the ratio 365/260 in a collection of *n* ratios is given by Bernoulli's law

$$P_n(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{(n-x)} \tag{1}$$

where *p* is the portion of possible ratios that we consider an identity with 365/260. The smaller we define *p*, the higher the accuracy we require before we accept a ratio as an identity with 365/260: as *p* increases we expect to find more identities. For large *n* as in our case we can run into problems with very large numbers, but fortunately as *n* gets larger the de Moivre-LaPlace theorem shows that the Bernoullii distribution approaches a normal distribution, and can be approximated to high accuracy by

$$P_n(x) = \frac{1}{\sqrt{2\pi np(1-p)}} e^{-\frac{(x-np)^2}{2np(1-p)}} \tag{2}$$

We can take *p* as the likely error made by the builders in laying out the separation between stones. As a first guess we might allow the builders something like a 1% error margin (we will find some justification for this sort of value below). In Figure 4, “Probability of finding the ratio 365/260” the probability of finding *x* identities in 630 ratios is shown for error allowances from 0.5 to 3.0% (p=0.005 to 0.03). The height of a curve at any value of *x* is the probability of finding that number of identities by chance alone, the area under the curves is unity, corresponding to the sum all possible identities. Although the distribution functions are presented as continuous, the reality of course is that fractional occurrences are impossible. This is illustrated by the points marked for each possible solution on the curve for an acceptable error of 3%. Also marked on the figure by vertical lines are the values found from the analysis of the observed ratios for acceptable errors of 1, 1.5, 2, and 3%, and is very obvious that it is highly improbable that these could be the result of chance alone, they all occur a long way down the tail where the probability curve is very close to zero. We must conclude that there was some constraint (intention) upon the part of the builders to favor the ratio 365/260. Statistically this can be quantified by the number of standard deviations, σ, of the observed value away from the mean (the peak of the curves). The observed and chance calculations are compared in Table 7, “Comparison of the Ratios 365/260 Found at Stonehenge 3 I with the Bernoulli Prediction for the Basic 9-point Geometry”. For example, for a 1.5% acceptable error the probability of finding the observed number of identities (25) is 5.1 standard deviations (sigmas) greater than the mean predicted by chance (9.45). The odds against

[xiv]The number of ways of choosing *x* objects out of a collection of *n* without any respect of order is known as a combination, given by

$$\frac{n!}{x!(n-x)!}$$

where *n!* is the factorial of *n*, i.e. *n*×(*n*-1)×(*n*-2)×...1

a 5.1 sigma event happening by chance are rather more than a million to one (10946491.2 to 1).[xv] It is noteworthy that for accuracies less than 1% the number of observed identities is much closer to the value expected by chance, only 1.6-1.7 standard deviations from the mean. This strongly suggests that the building errors in measuring lengths were actually of the order of 1% or greater, this would be of the order of plus or minus one stride in a hundred if the placement was carried out by pacing.

Figure 4. Probability of finding the ratio 365/260

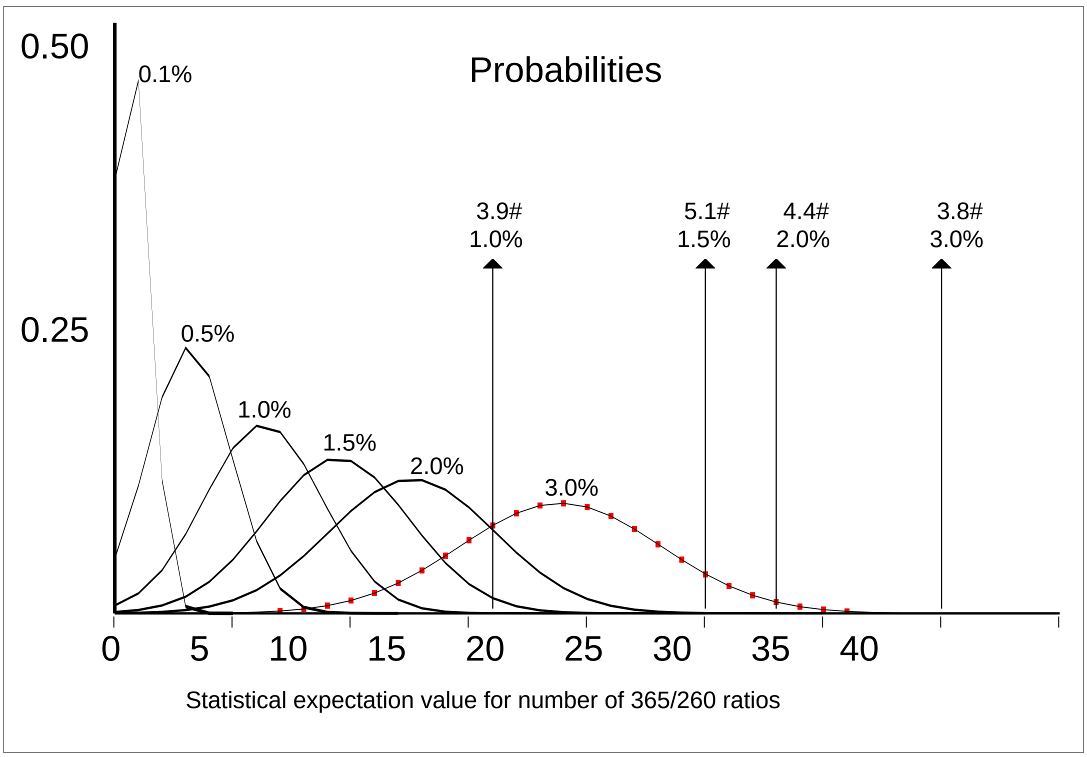


Table 7. Comparison of the Ratios 365/260 Found at Stonehenge 3 I with the Bernoulli Prediction for the Basic 9-point Geometry

[xv]A 5 sigma observation is accepted in particle physics at CERN as a certainty

Accuracy (%)	Found	Mean	standard deviation	sigmas	probability
0.1	2	0.6230	0.7933	1.72970	0.089797061
0.5	6	3.1500	1.7704	1.6098	0.109189917
1.0	16	6.3000	2.4974	3.8840	0.000211419
1.5	25	9.4500	3.0509	5.0969	0.000000912
2.0	28	12.600	3.5140	4.3825	0.000026935
3.0	35	18.900	4.2817	3.7602	0.000339346

This 1% error estimate is somewhat higher than that noted by Peeler and Winter at Teotihuacan (0.056%), but that accuracy was a measure of their construction rather than direct observation– There is no clear point in Tlailotlacan from which to measure distances, the point they used was dependent upon the *magic angle of 36°* that they propose was brought to Teotihuacan from Monte Albaán. For Building J at Monte albán the error was 0.077%, and for the ball courts 0.17%. It must be noted that only the last, the sizes of the ball courts, is the result of direct measurement. In the case of Building J, the measurements are of a hypothetical conjecture based upon the angular orientation of two sides of the building.

When we extended the analysis to include all the isolated standing stones and major stone holes which Hawkins has attributed to Stonehenge 3 I (A, B,and C associated with the Avenue to the Heel Stone, and D, E, F, G, and H lying close to the circle of the Aubrey Holes and the Station Stones), the 17 points thus defined generate 136 different lines, with a total of 9180 pairwise ratios. Within an accuracy of 1% we find agreement with the 365/260 ratio in 88 cases, with the best agreement being within 0.02%, an accuracy of magnitude close to those reported by Peeler and Winter. At first sight it might appear that this high number of ratios clustering close to 365/260 strongly suggests a highly improbable chance event. There is sufficient data for reliable analysis, and the statistics in Table 8, "Comparison of the Ratios 365/260 Found at Stonehenge 3 I with the Bernoulli Prediction for the Expanded 17-point Geometry Including Stones A–H", generated using the Bernoulli formalism, are perhaps somewhat surprising. At all levels of accuracy from 0.05 to 3% the numbers of identities found are close to the values predicted by chance. For a 1% error, chance predicts 91.8 identities with a standard deviation of 9.5, compared with the observation of 88. Clearly there is no evidence here for deliberate design, and we must question the attribution of holes A,B–H to Stonehenge 3 I. I have found no evidence to support the attribution of these holes to Stonehenge 3 I, although they did add considerable support to the astronomical alignments noted by Hawkins.

Table 8. Comparison of the Ratios 365/260 Found at Stonehenge 3 I with the Bernoulli Prediction for the Expanded 17-point Geometry Including Stones A–H

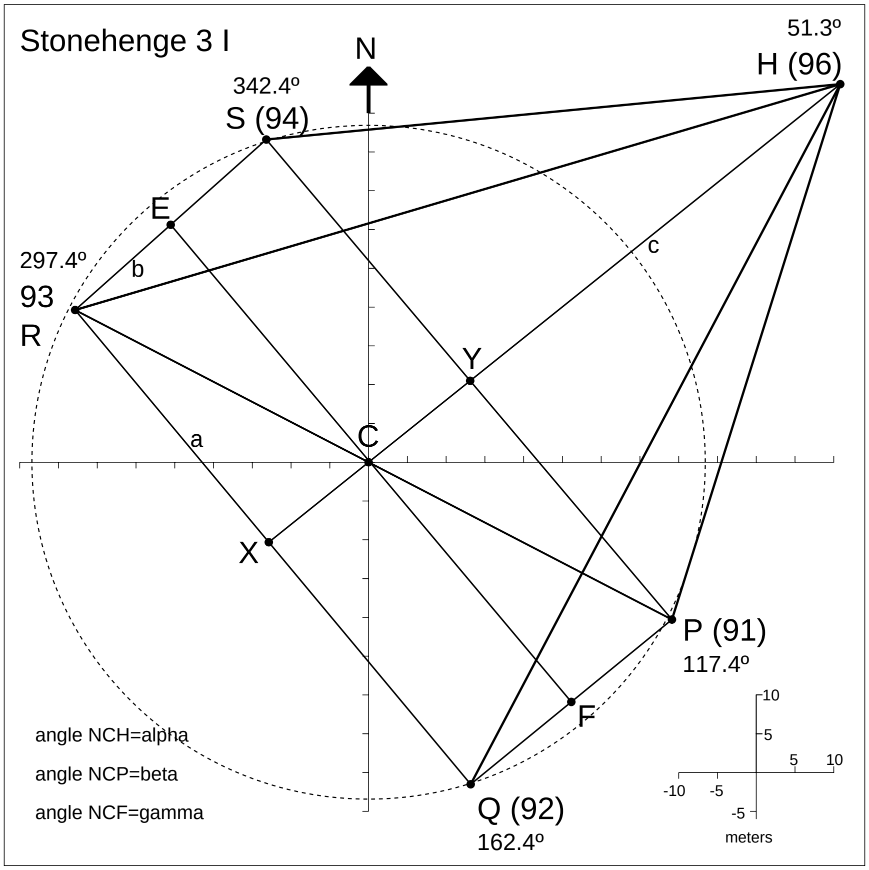
Accuracy (%)	Found	Mean	standard deviation
0.05	3	4.59	2.142
0.10	13	9.18	3.028
0.25	26	22.95	4.785
0.50	42	45.90	6.758
0.75	67	68.85	8.266
1.0	88	91.80	9.533
1.25	114	114.75	10.645
1.50	133	137.7	11.646
2.00	163	183.6	13.414
3.00	232	275.4	16.344

We have deliberately excluded from the analysis the locations of any of the stones of the complete circles at Stonehenge: the great Sarsen circle (30 of them around a circle of radius 14.9m), the Z holes (30 of them around a circle of radius 19.4m), the Y holes (30 of them around a circle of radius 25.8m). These are all approximately equally spaced and radially arranged. We have also excluded from the analysis the locations of the Aubrey holes (56 of them around a circle of radius 43.4m). There are no obviously significant locations on these circles, and ratios could be found among their distances corresponding to any value one cares to choose. It has been claimed that theses circles could have used as a computational device aiding the prediction of eclipses but it is also possible that they were simply cosmetic additions designed to impress onlookers.

Stonehenge may have the appearance of a very complex site today, but the initial structure, Stonehenge 3 I, was remarkably simple, Figure 3, "Basic Geometry of Stonehenge 3 I". Four stones mark out the three extreme rising positions of the sun and moon. The four Station Stones mark a rectangle that has its short sides pointing to the midsummer sunrise at 51.3° east of north. The long sides point to the midsummer moon-rise at 140.7°, while the diagonal at 117.4° points to the second midsummer moonrise.[xvi]

[xvi]In the 18.61 year cycle of the moon there are two extreme angular positions for the midsummer moonrise. the long side of the rectangle points to one, the diagonal to the other.

Figure 5. The Lengths of the Rectangle and the Distance of the Heel Stone



The sole remaining parameter is a size scaling length, and if this is taken to be the radius of the circle of Aubrey holes, then the whole rectangle with two lengths, (*PQ*, 91-92)=(*RS*, 93-94) and (*QR*, 92-93)=(*PS*, 94-91) is uniquely determined by the summer sun and moon rises together with the radius of the Aubrey Hole circle.

A knowledge of either of the angles beta or gamma is sufficient to determine the structure, so we have two independent solutions which should agree. For the purpose of the construction, we define two additional points, *Y* the midpoint of *PR*, and *X*, the midpoint of *QR* which both lie on *CH*. We calculate the parameters *a*=QR/2, *b*=RS/2, and *c*=*HY*. A Euclidean description follows.

$$a = r\cos(\beta - \alpha) \tag{3}$$

$$b = r\sin(90 - \beta + \alpha) \tag{4}$$

Alternatively

$$a = r\cos(\gamma - \beta) \tag{5}$$

$$b = r\sin(\gamma - \beta) \tag{6}$$

If we accept that the basic structure of Stonehenge 3 I was indeed laid out with the intention of a rectangle marking the critical points of the Sun and Moon (the angular appearances of the sun and moon at their extreme positions), then the positioning of the Station Stones is dependent on a single scaling parameter which completely determines the geometry of the rectangle.[xvii]

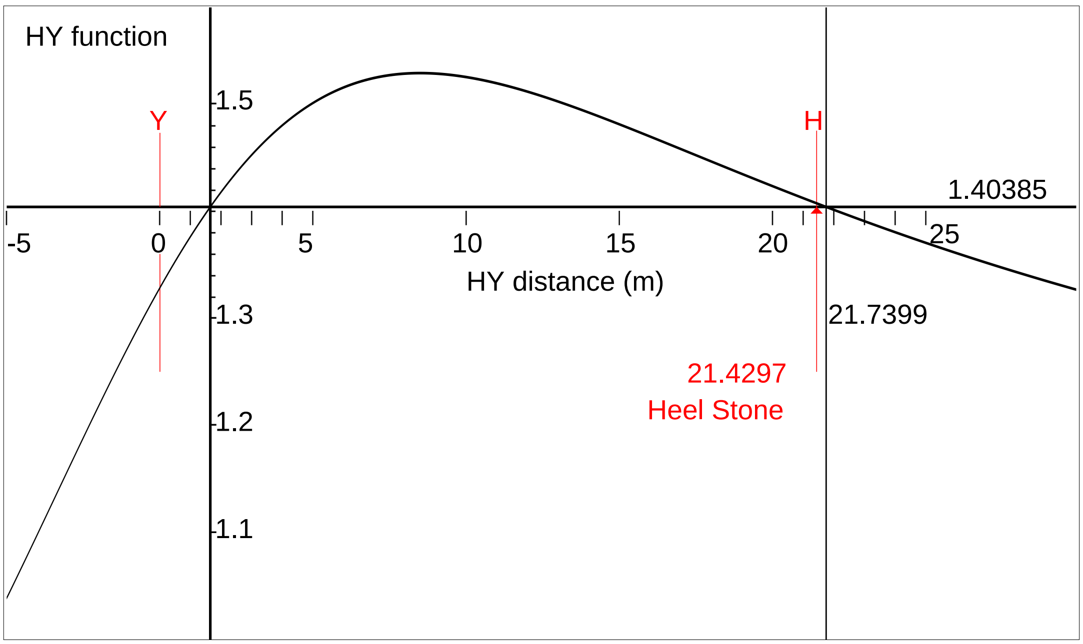
The adherence to the 365/260 ratio places severe constraints upon the location of the Heel Stone (*H*, 96) along the direction of the mid-summer sunrise. The distance of the Heel Stone from the Station Stone rectangle (*c*=*HY*) is then given by

$$c = \sqrt{(1 + \frac{4(bx + b^2)}{a^2 + x^2}} \qquad (7)$$

If we now add the additional requirement that *HR/HS*=365/260, then we need to find the value of *c* which yields a function value (*y*) = 365/260. The function in Equation 7 plotted in Figure 6, "HY, the distance of the Heel Stone from the rectangle of Station Stones" shows that there are two values of *HY* for which the function takes the value 365/260=1.403846154. One is very close to the rectangle and some 20m,within the Aubrey circle at *c*=*HY*=2.47283 which is clearly not an acceptable solution. The second solution lies some 20m outside the circle at *c*=*HY*=21.7399. This location for the Heel Stone is very close to that observed, 21.43m, providing further evidence that the structure of Stonehenge 3 I was deliberately constructed with the ratio 365/260 in mind, but with an error in placing the stone of 32cm (1.45%). If we could suppose that they were aware of a better approximation to the year's length was 365.2422 days, then the error in placing the Heel Stone would have been only 22cm (1.05%)–but this is pushing the data beyond their limits of accuracy. Again, an accuracy of 1-1.5% in placing stones seems very reasonable.

[xvii]This is certainly not meant to claim that the people responsible for the structure of Stonehenge 3 I were capable of this exercise of Euclidean geometry–it is far more likely that they proceeded by an iterative trial and error approach.

Figure 6. HY, the distance of the Heel Stone from the rectangle of Station Stones



To summarize, we have demonstrated that the *relative* positions and orientations of the Station Stones and the Heel Stone of Stonehenge 3 I can be uniquely determined by the three angles, α=51°18', β=117°4', and γ=162°4', all of which are determined by the critical sunrise and moonrise passages at mid-summer at the latitude of Stonehenge and the period of its construction, the length of the solar year in days (365), and if and only if the 'magic' number 260 is included in the description. Furthermore, the *absolute (actual)* locations are finally determined by the single distance scaling parameter, the radius of the Aubrey Holes which pre-dated the Station Stones.

7. The 260 problem

The solar year of 365 days has very obvious significance over the entire globe, but the length of the ritual calendar of 260 days has only been established in Mesoamerica. The question raised by Peeler and Winter is whether the ratio of these two numbers was deliberately used in architectural design and construction at Teotihuacan, and I believe we have to conclude that the evidence for the ratio 365/260 found at Stonehenge is very significantly stronger then that for Teotihuacan. In view of the absence of any cultural contact between these sites, the similarity is, to say the least, surprising.

There remains the problem of the significance of the number 260 upon which the ratio depends. As Peeler and Winter say "There is no general agreement as to why a period of 260 days was chosen as the ritual calendar". They list three possible explanations: the period (262 days) between zenith passages of the sun over the winter period, August 12 to May 1 at Izapa in Chiapas (latitude 14°54') considerably farther south than either Monte Albán (17°3') or Teotihuacan (19°41') (and also Stonehenge at 51°10'44"); the human gestation period (a best average of 266 days from conception); the product of 13 numbered days and 20 named days in Mesoamerican culture (hardly an explanation). Peeler and Winter add a fourth possibility: atan(260/365)=35.4634°, which is fairly but not convincingly close to the angle between the two zenith passages of the sun at the latitude of Monte Albán. Of these four possibilities, the first and last are not applicable at the latitude of Stonehenge. There is no evidence that the neolithic inhabitants of the Stonehenge region did or did not employ 13 or 20 in the numbering or naming of days, and it would have had to have been an surprising coincidence if they had happened to choose the same calendar as the Mesoamericans. The second, the human gestation period, is at best only approximate to within a few days, and also depends upon the observation and recognition of a day on which nothing visible happens.

Noting that humans have 20 digits, one possible approach to the origin of the 20×13=260 problem is to search for architectural expressions of the ratio 20/13=1.538461538.\, and some dimensions of Stonehenge structures display this ratio. The ratio of the length of the main axis from the Heel stone to the opposite side of the Aubrey circle (HT=42.30m in Figure 3, “Basic Geometry of Stonehenge 3 I”) to the distance between the Station Stones perpendicular to the main axis (PS=27.65m, giving a ratio of 1.530 and QR=27.60m, at 1.533 in Figure 5, “The Lengths of the Rectangle and the Distance of the Heel Stone”) is indeed very close to 20/13=1.538. Inside the outer ditch the structures fit neatly into a rectangle 121.4502m by 79.3159m, a ratio of 1.5312, about 0.47% from the ratio 20/13.

Another possibility is an astronomical origin for 13. We note that 5 synodic periods of Venus (2920 days) coincide with 8 solar years: 8+5=13. We explore this in more detail and for the moon and for the five planets, Mercury, Venus, Mars, Jupiter, and Saturn (with apologies to Uranus, Neptune and the ex-planet Pluto). The parameters for the planets and the moon are listed in Table 9, “Periods of the Planets and Moon”. The parameters of most interest are the periods, the synodic period as observed from earth, and the true orbital periods of which we concentrate on the tropical period as the most relevant to observers on earth.[xviii]

Table 9. Periods of the Planets and Moon

Planet	Visual Magnitude	Synodic Period	Sidereal Period	Tropical Period
Mercury	-0.42	115.88	87.060	87.968
Venus	-4.40	583.923	224.701	224.695
Earth	-3.86		365.24218408	365.24218408
Mars	-1.52	779.94	686.980	686.973
Jupiter	-9.40	398.88	4332.589	4330.595
Saturn	-8.88	378.09	10759.22	10746.94
Moon	+0.21	29.53		27.3217

Observers on earth may approximate the solar year to an integral number of days or to however accurate their observations happen to be. We start from the best estimates of the mean solar year and the periods of the planets and the moon, and calculate ratios of periods relative to the true solar year. In ??? we see that there are clearly two sets of ratios, those close to integral, differing by 0.05 or less, and those far from integral, differing by more than 0.2.

[xviii]The synodic year is the true orbital period. The sidereal period is the time between two successive observations of the same configuration as seen from earth. The tropical period is the elapsed time between two passages at right ascension zero (right ascension is the celestial equivalent of terrestrial longitude).

Table 10. Periods of the Planets and Moon

Planet	Period	length (days)	Multiplier	Product	* for integer
Mercury	Tropical>	87.968	33.2159	2921.937472	
Mercury	Synodic	115.88	25.2152	2921.937472	
Venus	Tropical	224.69526222	13.00400126	2921.937472	*
Venus	Synodic	583.923	5.00397736	2921.937472	*
Earth	Tropical	365.24218408	8	2921.937472	*
Mars	Tropical	686.973	4.2535	29021.9375	
Mars	Synodic	779.94	3.7464	2921.937472	
Jupiter	Tropical	4330.595	0.67472	2921.937472	
Jupiter	Synodic	398.88	7.3254	2921.937472	
Saturn	Tropical	10746.94	0.27189	2921.937472	
Saturn	Synodic	378.09	7.7282	2921.937472	
Moon	Tropical	27.3217	106.945669	2921.937472	*
Moon	Synodic	29.530588853	98.946130	* 2921.937472	

The ratio of the tropical period of Venus to the solar year is effectively the integer 13 we are looking for, and that 99 synodic periods of the moon (2923.47 days) is reasonably close to five solar years (2921.94 days). Perhaps optimistically we note that 99/5 is close to 20. We speculate that this may be an astronomical origin of the 260 day period, and that it is independent of latitude, so would apply equally well in Mesoamerica and Stonehenge (see Section 10, "*Venus, Mars, and Jupiter*" for a more detailed analysis of the ratios at Stonehenge).

Accordingly we suggest that the 260 day ritual calendar in Mesoamerica arose from astronomical observations of Venus.

8. Unit of Length, the Stride

Peeler and Winter note that "It is important to stress that we have found no Monte Albán 'meter,' ... It is the ratio, the proportion,that is significant, not the unit of measurement." I believe we are justified in going a little further into the unit of measurement.

An analysis of 64 megalithic sites in England and Scotland lead Thom,[THOM55]" to propose that a unit of 5.435ft. (165.6588cm) was evident in the dimensions of these structures with a probability of chance between 0.001 and 0.005. This unit is close to a double stride of 0.8283m[xix] Surprisingly Thom did not include Stonehenge among his sites, but Thom was primarily interested in deviations from circularity of the monuments he studied, and there are no significant deviations from circularity at Stonehenge. If we assume that the neolithic people at Stonehenge could count, but were not aware of any fractional system, then we should be able to find a unit of length that gives integral counts for all the lengths 96-92, 96-93, 96-91, and 96-94, and from the center to 91, 92, 93, and 94, assuming bilateral symmetry was the structural intention. Restricting the choice of a stride length to between 0.6 and 0.9m leads to a very sensitive choice of a single value of 0.82975m, giving 106.000 strides for 96-91 and 96-94, 149.008 strides for 96-92 and 96-93, and 22.027 strides for the center to 91, 92, 93, and 94. This stride length is remarkably close to that estimated by Thom (0.82829m) from 64 neolithic constructions in what are now England and Scotland.

Searching for a similar integral number of strides at Monte Albán and Teotihuacan yields good agreement for a stride length of 0.7651 for the six lengths quoted by Peeler and Winter: Table 11, "Stride1 table" for example, *QM*=1964.7m in 2367.1 strides (0.000465%), *TQ*=2756.6m in 3321.2 strides (0.006022%). The average error over all 6 lengths is only 0.195%, xref linkend="Stride_table2"/>. Can we assume that long lengths in Mesoamerica were measured by strides?

Table 11. Stride1 table

Measured length	Calculated length	Difference	Percent Difference	Number of 0.7651m strides
1964.70	1964.92	-0.22	0.00	3535.2
2756.60	2756.74	-0.14	0.00	4960.1
40.67	40.85	-0.18	0.13	73.2
29.02	29.23	-0.21	0.23	52.2
77.25	77.25	-0.00	0.00	139.0
55.07	55.16	-0.09	0.05	99.1

[xix] At my height of 6ft (1.83m), my stride on average is about 0.946m, but there is clear evidence that the height of the neolithic population of England was considerably less than mine.

Table 12. Stride Length Summary

	Teotihuacan QM	Teotihuacan TQ	Ball Court 1	Ball Court 2	Building J 1	Building J 2
Measured length	1964.70	2756.60	40.67	29.02	77.25	55.070
Stride length	1964.92	2756.74	40.85	29.23	77.25	55.161
Difference	-0.22	-0.14	-0.18	-0.21	-0.00	-0.091

Perhaps the illustration of two men measuring with a rope in the Codex Vindobonensis Anverse is only applicable to short distances. We need to address the problem of how the Zapotecs could have measured distances longer than a kilometer. Certainly they would not have measuring rods or ropes much longer than a few meters, and cumulative errors in applying measuring rods to such long distances would have resulted lengths bearing no relation to design. In contrast, counting strides naturally compensates for any variation in stride length, and improves accuracy with overall distance. The stride is a natural and convenient measure, although it would have been easy to lose count over long distances.

Finally, we might estimate the height of an Mesoamerican strider to be about 1.479m (4ft.10in.) by scaling from my height of 1.83m and stride of 0.9464m. This looks eminently reasonable. For Stonehenge the height of the strider would be 1.60m (5ft.3in.), some 5 inches taller than his Mesoamerican cousin. This could perhaps be checked against skeletal remains.

9. The Steps of the North Platform at Monte Albán

Peeler and Winter also noted that the ratio of the widths of the steps at the North Platform at Monte Albán was close to the ratio 584/365, the ratio of the synodic period of Venus to the solar year. The same search for integral ratios was carried out for this system, and, perhaps not surprisingly, the same result is shown in Table 13, "The Width of the Steps of the North Platform at Monte Albán"

Table 13. The Width of the Steps of the North Platform at Monte Albán

delta (δ)	Number of ratios found	Numerator	Denominator	Error (%)
0.0000149491	0			
0.0000149492	1	497	311	+0.00093546
0.000017	2	163	102	-0.00103718
0.000031	3	334	209	+0.00189818
0.000041	4	644	403	-0.00255949
0.000046	5	505	316	+0.00284567
0.000050	6	481	301	-0.00307535
0.000066	7	318	199	-0.00412004
0.000076	8	171	107	+0.00469638
0.000083	9	473	296	-0.00518238
0.000092	10	628	393	-0.00572031
0.000104	11	521	326	+0.00649031
0.000118	13	155	97	-0.00736182
		350	219	+0.00736680
0.000132	14	529	331	+0.00823005
0.000145	15	612	383	-0.00904619
0.000200	21	366	229	+0.01235781
0.000300	31	286	179	-0.01817379
0.000400	43	414	259	+0.02501841
0.000500	53	131	82	-0.03096355
0.000600	64	377	236	-0.03743069

0.000700	74	484	303	-0.04355633
0.000800	87	558	349	+0.04994055
0.000900	97	337	211	-0.05628040
0.001000	109	543	340	-0.06238623
0.001500	170	372	233	-0.09319241
0.001944	192	83	52	-0.11901613
0.001945	193	8 (584)	5 (365)	+0.12166094

Again, there is no identity with an integral ratio with an error less than 0.00093546%, and there are 192 ratios in better agreement than 8/5=584/365, with the best one having an error 130 times less than the Venus ratio. Peeler and Winter also noted two other (closely related) ratios at Teotihuacan approximating to the 584/365 ratio, but gave no details of measurements.

Again, we see the identity of the ratio of lengths with 584/365 at Monte Albán is most likely not only coincidence, but a poor coincidence. The Venus ratio does not appear to be particular significant at Stonehenge, but there is evidence for the synodic period of Jupiter in the layout of stones at that site. We examine the planetary ratios at Stonehenge more closely in the following section.

10. Venus, Mars, and Jupiter

Our statistical approach to the 365/260 ratio at Stonehenge is strongly indicative of a deliberate design intention from a very early period. The design is built into the first stones that were laid out on the ground. The probability of a chance placing of the stones such that the ratio of the distances were close to the 365/260 ratio was in the region of a 4-5σ event–the odds against this happening by accident or chance are around one million to one. It is difficult to see just how the placement of these stones could have resulted from an evolutionary or trial-and-error process.

Including the stones A, B, ... H into the analysis destroyed the uniqueness of the placement of the first stones, and emphasised the human intentions in the design of the first structure.

However, the investigation above has been restricted to the existence of a single ratio, 365/260, which needs to be tested by checking other ratios. Two possibilities are obvious: firstly a random choice of two integers (not too close to 365/260), and secondly an extended search for other ratios of astronomical significance. The first is expected to result in statistics close to those predicted by the Bernoulli formalism, the second would either mimic the first, or might possibly indicate another significant (and astronomical) ratio built in by design. As expected, a search for the ratio 456/123 yielded identity counts ranging from 2 at 0.1% accuracy to 8 at 3.0%, all rather lower that expected from the Bernoulli distribution.

Apart from the moon, the brightest and most mobile lights against the background of stars in the night sky are the planets Venus, Mars, and Jupiter, and we include here a preliminary examination of the ratios created from the synodic periods of these planets.[xx] Fortunately the synodic periods of the three planets are sufficiently different to produce a wide range of ratios to test. In Table 14, "Venus, Mars, and Jupiter Ratios" we present the numbers of identities found in Stonehenge 3 I, and include the corresponding values for 365/260 and Bernoulli expectation values for easy comparison. A quick examination of the table suggests that ratios involving the synodic period of Jupiter with both 365 and 260 are very significantly different from Bernoulli expectation values. The synodic period of Mars (780) does not appear to correlate with either Earth (365) or ritual (260), the values found are close to Bernoulli expectations. Correlations of Mars (780) with Venus (584) and Jupiter (399), and Venus (584) with Earth (365) and Jupiter (399) might possibly be significant. This is somewhat surprising as Venus is by far the brightest light in the night sky after the moon, and yielded such good approximations to the integers 5 and 13 (see Table 10, "Periods of the Planets and Moon".

Table 14. Venus, Mars, and Jupiter Ratios

[xx]The synodic periods are those observed on earth between appearances of a planet at the same location in the sky, easiest to measure as a rising or setting event. It is important to avoid confusion with the sidereal period of the planet which is the time taken for a complete revolution around the sun. These period can be very different, for Venus the sidereal period is 244.62 days, but the synodic period is 584 days (see Table 9, "Periods of the Planets and Moon".

Greater Synodic Period	Lesser Synodic Period	0.5%	1.0%	1.5%	2.0%	2.5%
780	260	8	9	10	13	13
780	365	5	8	9	12	16
780	399	4	11	20	22	20
780	584	4	7	15	22	27
584	260	3	6	10	13	15
584	365	1	10	17	21	26
584	399	6	12	16	16	26
399	260	6	19	23	27	35
399	365	12	19	24	33	37
365	260	6	16	25	28	31
Bernoulli	Expectation	3.15	6.30	9.45	12.60	15.65

If we now concentrate on the ratios involving Jupiter in Table 15, "Jupiter Ratios" we see that the ratios involving Venus (584) and Mars (780) are less convincing than those involving earth (365 and 260).

Table 15. Jupiter Ratios

Planet	Synodic Period of Jupiter	Synodic Period	0.5%	1.0%	1.5%	2.0%	2.5%
Mars	399	780	4	11	20	22	29
Venus	399	584	6	12	16	16	26
Earth	399	365	12	19	24	33	37
Ritual	399	260	6	19	23	27	35
	365	260	6	16	25	28	31

[xxi]There is no possibility of overlap confusion between the 399/260=1.535 and the 365/260=1.446 ratios within the 3% error limit, the closest approach being 1.489 to 1.446

Focussing on the Jupiter earth ratios we find the probabilities of finding these results by chance alone are very low indeed, all in the four to five sigma range Table 16, "Sigma Values for Jupiter Ratios".[xxi]

Table 16. Sigma Values for Jupiter Ratios

Planet	Planet	1.0%	1.5%	2.0%
Jupiter (399)	Earth (365)	5.1	4.8	5.8
Jupiter (399)	Ritual (260)	5.1	4.5	4.1
Earth (365)	Ritual (260)	3.9	5.1	4.4

Provisionally therefore I believe we must include observations of the synodic period of Jupiter in the design of Stonehenge. However the ratios with Venus and Mars have probabilities in the two to three sigma range, and may repay further investigation.[xxii]

[xxii] I rely upon past experience at CERN, a 3σ event is only a *definite maybe*, whilst a 5σ event can be regarded as a certainty.

11. Conclusions

- The ratio 365/260 in the configuration of the first stones laid at Stonehenge (2600BCE) is confirmed to a very high degree of statistical confidence–a 5-σ event. This indicates that the Mesoamerican ritual calendar was based upon factors independent of latitude.
- The ratios 359/256, 355/253, 379/270, 369/263, all close to 365/260, are common to the three sites in México and exhibit better fits to the observations than 365/260. A total of 50 ratios of integers were common to the three Mexican sites, all in better agreement with measurements on the ground, than 365/260.
- The importance of the 260 day period at both Mesoamerican sites and at Stonehenge confirm that this period is independent of latitude, cultural influence, and historical period.
- The origin of the 260 day period probably resides in astronomical observations of both the tropical and synodic periods of the moon and Venus.
- There is strong evidence that astronomical observations of Jupiter were also included in the layout of stones in the earliest Stonehenge.

The computer codes (Perl 5.12.3 run under Windows 7) used in this study are freely available, without any guarantee of correctness or usefulness, from the author by email request to ron@catterall.net

References

[HAWKINS63] Gerald S Hawkins. *Stonehenge Decoded*. Nature. 200. 306-308. (1963).

[HAWKINS65] Gerald S Hawkins and John B White. *Stonehenge Decoded*. (1965). Souvenir Press (Doubleday & Co. Inc in USA). London1965.

[NEWALL65] R A Newall. *Stonehenge*. Wiltshire. 4th amendment. (1965). Her Majesty's Stationary Office. London.

[THOM55] Alexander Thom. *A Statistical Examination of the Megalithic Sites in Britain*. Journal of the Royal Statistical Society, Part III. CXVII. 275-295. (1955).

[SUNPOS] http://www.sunearthtools.com/dp/tools/pos_sun.php.

[WINTER95] Damon E Peeler and Marcus Winter. *Buiilding J at Monte Albn:*. A Correction and reassessment of the Astronomical Hypothesis. Latin American Antiquity. 6(4). 362-369. (1995).

[WINTER10] Damon E Peeler and Marcus Winter. *Sun Above, Sun Below*. Astronomy, Calendar and Architecture at Monte Albn and Teotihuacan. First. . (November 2010). Centro INAH, Oaxaca, Arqueologia Oaxaquea, Serie Popular. Pino Surez 715, 68000 Oaxaca, Oaxaca, Mexico.